INVESTIGATION OF TIME DEPENDENT EFFECTS ON COMPOSITE BRIDGES WITH PRECAST INVERTED T-BEAMS

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ABSTRACT

This paper investigates time dependent effects on composite bridges with precast inverted T-beams. The analysis is performed for a two-span continuous bridge. This system provides enhanced performance against reflecting cracking because it offers a thicker cast-in-place topping over the joint between the precast members. An analytical study is performed to quantify the stresses generated as a result of differential shrinkage, creep and temperature gradient at various sections in both directions. At the crosssectional level, an elastic sectional analysis approach using the age adjusted effective modulus method is used to perform the investigation. At the structure level the effects of uniform temperature changes, thermal gradients and differential shrinkage and creep are investigated and quantified in terms of axial restraint forces and restraint moments. It is shown that by paying attention to detailing and by selecting a mix for the cast-in-place topping that has relatively low shrinkage and high creep the potential for excessive cracking can be reduced and the longevity of the bridge prolonged. Results are presented and recommendations are made for strategies to reduce the magnitude of tensile stresses created as a result of these effects.

Keywords: Precast, Shrinkage, Creep, Temperature, Excessive Cracking

INTRODUCTION

On March 19, 2013 the ASCE 2013 Report Card for America's Infrastructure was published and the United States received a $D+^1$. The Report Card provides a comprehensive assessment of the current infrastructure condition and needs by assigning grades and by making recommendations for improvements. According to the Report Card one in nine of the nation's bridges are rated as structurally deficient, while the average age of the nation's 607,380 bridges is 42 years. Relatively speaking bridges and railroads did better than the rest of the infrastructure by receiving a C+. The Federal Highway Administration (FHWA) estimates that to eliminate the nation's bridge deficient backlog by 2028, United States would need to invest \$20.5 billion annually, while only \$12.8 billion is being spent currently.

While many engineers design for the effects of dead, live, seismic and wind loads, not many consider the effects of shrinkage, creep and temperature as significant. Load cases including these effects can in certain bridge types lead to tensile stresses in excess of the tensile strength of concrete. Good examples are composite concrete bridges that consist of precast and cast-in-place elements. To accelerate bridge construction the typical construction sequence for a composite concrete bridge entails the precast elements to be cast long before the cast-in-place topping is placed. This sequence creates a more pronounced difference in the shrinkage and creep properties of the precast and cast-in-place elements because these properties are time dependent.

Short to medium span bridges can be constructed using precast voided slabs or adjacent box girders finished with a cast-in-place topping. This type of construction has manifested longitudinal reflective cracking along the girder-to-girder interface. To address this problem, a new bridge system has been proposed: precast inverted T-beams with a castin-place topping. This system was identified during a scanning tour in Europe and Japan funded by the Federal Highway Administration (FHWA)². The state of Minnesota was the first state in US to implement this system in several bridges^{3,4,5,6,7,8}. Virginia will build its first inverted T-beam bridge in 2014 on U.S.360 over Chickahominy River near Richmond, VA. The research team at Virginia Tech built upon the findings and experiences of the state of Minnesota and made several recommendations optimizing the cross-sectional shape and the inverted T-beam to inverted T-beam connection⁹. This paper provides the results of a time dependent analysis for the two span continuous bridge that will be built in Virginia by considering the effects of differential shrinkage, creep and temperature. The objective was to investigate the effect of time dependent properties of the concrete topping, cross-sectional shape of the precast inverted T-beams, amount of mild steel in the deck, boundary conditions at the abutments and age of continuity on the magnitude of tensile stresses in the composite cross-section in the longitudinal and transverse directions. Controlling the magnitude of these tensile stresses is important to avoid excessive transverse and longitudinal cracks.

IMPLEMENTATION OF THE INVERTED T-BEAM SYSTEM IN VIRGINIA

Implementation of the inverted T-beam system will occur in the U.S.360 Bridge over Chickahominy River. Figure 1 shows an elevation view of the two-span river crossing. Figure 2 shows a transverse cross-section of the bridge. The bridge has two equal spans of 43 ft. and has an approximate width of 110 ft.



Fig.1 Elevation view of the U.S.360 Bridge (Courtesy of VDOT)



Fig.2 Transverse cross-section of the bridge.



Fig.3 Reinforcing details for a typical 6'-0" section of the bridge

The depth of the precast inverted T-beams is 18 in. and the depth of the cast-in-place concrete topping over the web of the precast beams is 7 in. Design concrete strengths (f_c) for the precast inverted T-beams and cast-in-place topping are $f_c = 8000$ psi and $f_c = 4000$ psi, respectively. Figure 3 shows reinforcing details for a typical 6'-0" section of the bridge. The cross-sectional dimensions shown in Figure 2 and reinforcing details shown in Figure 3 represent one of the options considered by VDOT during the design phase and not necessarily the final design.

TIME DEPENDENT ANALYSIS

Bridges constructed with prefabricated elements offer many advantages over conventional construction methods. In many cases the precast components serve as stay-inplace formwork for the cast-in-place topping or deck. This eliminates the need to erect and remove formwork and results in shorter construction time and reduction in traffic disruption. However many existing bridges with precast components have durability issues, such as excessive cracking, which results in significant maintenance and replacement costs. This can eclipse the advantages that would otherwise be associated with these types of systems. Time dependent effects, such as differential shrinkage between the cast-in-place and precast components, are a major reason for the development of this cracking.

In conventional cast-in-place, shored construction, the self-weight of concrete will cause compressive stresses in the top surface of the deck in positive moment regions, and tensile stresses in negative moment regions. Additionally, tensile stresses created due to time dependent effects are limited to differential shrinkage between cast-in-place concrete and reinforcing steel and those created due to temperature gradients. Consequently potential cracking is limited to negative moment regions because elsewhere compressive stresses due to the self-weight of concrete counterbalance any tensile stresses created due to time dependent effects. The situation is different in systems that involve precast elements. Because the precast components provide support to the cast-in-place topping, the weight of the topping causes stress in the precast beams. As a result, the effects of differential shrinkage are more pronounced and can cause critical stress situations in the topping.

The following presents a time dependent analysis at the cross-sectional level as well as at the structural level to quantify stresses developed as a result of differential shrinkage, shrinkage induced creep, negative/positive temperature gradients and a uniform decrease in the temperature.

To promote a comfortable ride and to reduce the likelihood of leakage to the substructure many engineers design precast girder bridges as continuous for live loads. Continuity is provided by placing a cast-in-place concrete topping over the precast elements, which creates a continuity diaphragm at the interior supports. Additionally, reinforcing steel is provided to connect the bottom of the precast girders over interior supports. The age of the precast beams when this continuity is established plays an important role in the development of time dependent effects. The analysis performed in this paper assumes a precast girder age

of 90 days or more, before continuity is established. At this age most of the shrinkage and creep in the precast girder has occurred.

The advantage of specifying a high age for continuity is the reduction of positive restraint moments at the intermediate supports. These positive restraint moments may develop due to creep of the precast beam, as well as due to positive thermal gradients. These positive restraint moments can be high enough to overcome the effects of negative live load moments³. In addition, these positive restraint moments can also be high enough to render the positive moment connection over the piers as not providing 100% continuity.

One of the disadvantages of waiting for 90 days is that the differences in shrinkage and creep properties between the precast and cast-in-place components become more pronounced. Because the age of continuity for the bridge under consideration was assumed to be 90 days the ultimate shrinkage strain and creep coefficient for the precast girder were neglected. The corresponding values for the cast-in-place topping were taken as follows:

 $\epsilon_{\text{shdeck}} = -466 \text{ x } 10^{-6}$, $\phi_{\text{deck}} = 1.87$

These values were based on testing of four different concrete mixes with a design compressive strength at 28 days of $f'_c = 4000$ psi. The goal was to identify a mix with low shrinkage and high creep. The aggregates used in the four mixes consisted of normal weight and light weight aggregates. Additionally, the cementitious materials consisted of fly ash and slag. The ultimate shrinkage and creep coefficient values provided above represent the concrete mix with the lowest shrinkage and highest creep. The aging coefficient was assumed to be 0.7.

TIME DEPENDENT ANALYSIS AT THE CROSS-SECTIONAL LEVEL

Differential Shrinkage and Shrinkage Induced Creep

Regardless of the boundary conditions, the inherent difference in shrinkage and creep properties between the cast-in-place topping and precast girders will cause self-equilibrating stresses at the cross-sectional level. Even if the composite beam is used in a single span simply supported bridge, these self-equilibrating stresses will form along the entire span of the bridge. The difference in shrinkage properties is exacerbated by the difference in age between the two components. As a result when the topping is placed, it will tend to shrink while the majority of the shrinkage in the precast component has already taken place. The restraint provided by the precast component to the free shrinkage of the deck will create a tensile force in the deck while the free shrinkage of the deck will exert a compressive force in the precast beam.

In addition, because the centroids of the precast and cast-in-place components are at different locations, this differential shrinkage will cause a positive curvature. The curvature will result in a prestress gain in the bottom layer of prestressing in the precast beam, whereas the compression force from the shrinkage of the deck will cause a prestress loss. Another advantage of the precast inverted T-beam system is that the difference between the centroids of the cast-in-place and precast components is smaller compared to a similar voided slab or

adjacent box girder system. Consequently the curvature induced due to differential shrinkage is smaller.

Mild steel in the deck will provide an additional level of restraint against the free shrinkage of the deck and will therefore increase the tensile stresses in the concrete topping. Figure 4 shows the idealized locations of mild steel and prestressing steel used in the time-dependent analysis. The amount of mild steel and prestressing steel was based on the design of the U.S.360 Bridge per 2010 AASHTO LRFD Bridge Design Specifications¹¹.

The quantification of forces and stresses created due to differential shrinkage and shrinkage induced creep can be done using the principles of equilibrium, compatibility and material constitutive relationships. Menn¹⁰ provides detailed guidance on how this analysis can be performed. Some of the theoretical background provided in Menn's book¹⁰ is presented below for convenience. Figure 5 shows composite cross-section 2 and the change in strain and forces due to differential shrinkage and shrinkage induced creep. An elastic sectional analysis approach using the age-adjusted effective modulus method is used to calculate these changes. For example, the change in strain at the centroid of deck and girder can be determined by computing elastic and creep strains due to the change in axial force plus the strain due to free shrinkage (Equations 1 and 4). Similarly, the change in curvature can be determined by calculating elastic and creep curvatures due to the change in moment (Equations 2 and 5). The change in strain in any given steel layer can simply be determined by computing the elastic strain due to the change in axial force in the corresponding layer. In addition, because there are no externally applied axial forces or moments the sum of the change in axial forces and moments needs to be equal to zero (Equations 6 and 7). Assuming that there is a perfect bond between the cast-in-place deck, precast inverted T and mild steel, the axial strains at the centroid of each component can related by utilizing the curvature and the relative distances (principle of compatibility). Equation 8 provides one such example. By using Equations 1-8 a set of 15 equations and unknowns can be created and solved simultaneously. The unknowns would include changes in strain and forces in each component and the change in curvature. After solving for the unknowns, the change in stress at any given location in the precast inverted T-beam, deck or at any layer of mild steel can be calculated using Equations 9-11.



Section 3-Longitudinal Section through Precast Flange Fig.4 Idealized locations of mild steel and prestressing steel used in time dependent analysis



Fig.5 Forces in composite section 2 due to differential shrinkage and creep

$$\Delta \varepsilon_D = \frac{\Delta N_D}{E_D A_D} \left(1 + \mu \varphi_D \right) + \varepsilon_{SHD} \tag{1}$$

$$\Delta X = \frac{\Delta M_D}{E_D I_D} \left(1 + \mu \varphi_D \right) \tag{2}$$

$$\Delta \varepsilon_s = \frac{\Delta N_s}{E_s A_s} \tag{3}$$

$$\Delta \varepsilon_G = \frac{\Delta N_G}{E_G A_G} \left(1 + \mu \varphi_G \right) + \varepsilon_{SHG} \tag{4}$$

$$\Delta X = \frac{\Delta M_G}{E_G I_G} \left(1 + \mu \varphi_G\right) \tag{5}$$

$$\Delta N_D + \Delta N_G + \Delta N_{s1} + \Delta N_{s2} + \Delta N_{s3} + \Delta N_{s4} = 0$$
(6)

$$\Delta M_G + \Delta M_D - \Delta N_D a - \Delta N_{s1} a_{s1} - \Delta N_{s2} a_{s2} + \Delta N_{s3} a_{s3} + \Delta N_{s4} a_{s4} = 0$$
(7)

$$\Delta \varepsilon_D = \Delta \varepsilon_G - \Delta X \left(y_{Dbottom} - y_{Gbottom} \right) \tag{8}$$

$$\Delta \sigma_D = \left(\frac{\Delta N_D}{A_D} + \frac{(\Delta M_D)}{I_D} y\right) \tag{9}$$

$$\Delta\sigma_G = \left(\frac{\Delta N_G}{A_G} + \frac{(\Delta M_G)}{I_G}y\right) \tag{10}$$

$$\Delta\sigma_S = \left(\frac{\Delta N_S}{A_S}\right) \tag{11}$$

The assumptions made during this analysis are:

- Plane sections remain plane
- Sections are un-cracked
- Creep and shrinkage properties are assumed to represent the average behavior of the whole cross-sections, or components thereof, in drying conditions.
- Tensile creep is the same as compressive creep

Figure 6(a) shows the stress distributions in Sections 1, 2 and 3 caused by differential shrinkage and shrinkage induced creep. In this paper tensile stresses are positive and compressive stresses are negative. The stress distribution shown for Section 1 applies at the portion of this section where the thickness of the precast inverted T-beam is 18 in. and the thickness of the cast-in-place topping is 7 in. The maximum tensile stresses at the bottom of the cast-in-place topping in Sections 1, 2 and 3 are 0.37 ksi, 0.503 ksi and 0.487 ksi respectively. The modulus of rupture (f_r) for the deck is 0.474 ksi (based on $7.5\sqrt{f'c}$ where $f'_{c} = 4000$ psi). This highlights the potential of differential shrinkage to cause longitudinal cracking in the deck. The maximum tensile stresses at the bottom of the deck and at the bottom of precast inverted T-beam in Section 1 are lower than the ones in Section 2. As mentioned earlier this is due to the fact that the moment arm between the centroids of the cast-in-place topping and the precast beam in Section 1 is lower than in Section 2. This promotes the utilization of the inverted T-beam system as opposed to a voided slab system or adjacent box girder system, considering that Section 2 represents a similar section in the transverse direction in both of these systems, in addition to the longitudinal direction. The compressive stress at the top of the precast inverted T-beam is higher in Section 1 than in Section 2 due the higher volume of concrete. However, given that the weakness of the concrete is its tensile strength, this will not control design.

Figures 6 (b), (c) and (d) show the sensitivity of the tensile stress at the bottom of the deck to shrinkage and creep properties of the deck for Sections 1, 2 and 3, respectively. The horizontal and the vertical lines represent the modulus of rupture and the ultimate shrinkage strain for the deck, respectively. For example in Section 1 for a creep coefficient $\varphi=2$, there is a 78 psi decrease in the tensile stress for every 100 $\mu\epsilon$ decrease in the ultimate shrinkage strain of the topping mix (ϵ_{shdeck}). Similarly, for an ultimate shrinkage strain of $\epsilon_{shdeck} = -500 \times 10^{-6}$ there is a 119 psi decrease in the tensile stress for every increase by 0.5 in the creep coefficient. Clearly a mix with lower free shrinkage and high creep will be ideal from the standpoint of reducing tensile stresses as a result of differential shrinkage. High creep properties are desired to relieve the stresses developed as a result of differential shrinkage. Low shrinkage in the deck is desired to minimize the amount of differential shrinkage, provided that most of the shrinkage in the precast beam has already taken place.



Fig.6 (a) Stress distribution due to differential shrinkage and shrinkage induced creep in all three cross-sections; (b), (c) and (d) Sensitivity of tensile stress at the bottom of the deck to shrinkage and creep properties of the deck for Sections 1,2 and 3, respectively

The presence of mild steel in the deck restrains the free shrinkage of the deck and as a result creates additional tensile stresses. Figure 7 shows the sensitivity of the tensile stress at the bottom of the deck to the amount of mild steel. In Figure 7(a) A_{smild1} , A_{smild2} and A_{smild3} represent the variation in areas of mild steel in the deck in the longitudinal direction at different elevations. These are denoted as A_{s1} , A_{s2} and A_{s3} in Figure 4 - Section 1, respectively. In Figures 7(b) and 6(c) A_{smild1} and A_{smild2} represent the variation in areas of mild steel in the deck in the transverse direction at different elevations. These are denoted as A_{s1} and A_{s2} in Figure 4, Section 2 and Section 3, respectively. The vertical lines in Figure 6 represent the actual amounts of mild steel in the deck, which were based on the design of the U.S.360 Bridge per 2010 AASHTO LRFD Bridge Design Specifications¹¹. It can be seen that while the magnitude of the tensile stress at the bottom of the deck increases with an increase in the amount of mild steel, this increase is almost negligible. As a result mild steel needs to

be provided in the deck in both directions to control the width of potential cracks and it does not significantly increase the likelihood of cracking.





Temperature Gradient

Temperature gradients create similar effects to the ones created by differential shrinkage. Because temperature can vary through the depth of the cross-section, some parts of the cross-section will tend to contract or expand more than the other parts. The temperature gradient used in this study was obtained from the 2010 AASHTO LRFD Bridge Design Specifications¹¹ for the U.S.360 Bridge near Richmond, VA. The positive and negative temperature gradients have a bi-linear shape and are shown in Figure 8. Assuming plane sections remain plane, this bi-linear variation in temperature will cause self-

equilibrating stresses in the cross-section. These stresses can be calculated using the principles of equilibrium, compatibility and material constitutive relationships¹³. A sensitivity analysis for the creep and aging coefficients was not done because it was assumed that the temperature gradient would develop over a period of 8 hours. As a result the changes in creep and aging coefficients over such a short period of time would be negligible.





Some of the theoretical background presented in Gilbert¹³ for the calculation of selfequilibrating stresses due to thermal gradients is presented below for convenience. Figure 9 illustrates this approach by taking Section 2 as an example and the negative temperature gradient shown in Figure 8. If all the fibers in the composite cross-section were free to contract independently to accommodate the imposed negative temperature gradient, then the result would be the free strain diagram shown in Figure 9. The corresponding stress distribution can be calculated by simply multiplying the free strains with the moduli of elasticity of each component. However, because plane sections will tend to remain plane, the individual fibers will not be able to freely contract to accommodate the temperature gradient without violating the principle of compatibility. As a result, there will be some restrained stresses in the composite cross-section which are equal and opposite to the free stresses. The stress resultants of these restrained stresses (axial force and bending moment) can be calculated using Equations 12 and 13. Finally, the change in stress due to the imposed temperature gradient can be computed using Equations 14-16.



Fig.9 Approach for calculating self-equilibrating stresses due to thermal gradients (Section 2)

$$\Delta N = \int \alpha T(y) E b \, dy \tag{12}$$

$$\Delta M = \int \alpha T(y) Eby \, dy \tag{13}$$

$$\Delta \sigma_D = -\Delta \sigma_{Dfree} + \left(\frac{\Delta N}{A_{tr}} + \frac{\Delta M y}{I_{composite}}\right) n_D \tag{14}$$

$$\Delta \sigma_G = -\Delta \sigma_{Gfree} + \left(\frac{\Delta N}{A_{tr}} + \frac{\Delta M y}{I_{composite}}\right)$$
(15)

$$\Delta \sigma_s = -\Delta \sigma_{sfree} + \left(\frac{\Delta N}{A_{tr}} + \frac{\Delta M y}{I_{composite}}\right) n_s \tag{16}$$

Figure 10 shows stress distributions in Sections 1, 2 and 3 due to negative and positive temperature gradients. The largest negative temperature gradient tensile stress is at the top of the deck and is slightly higher than the largest tensile stress created as a result of a positive temperature gradient (0.15 ksi versus 0.11 ksi). These stresses are lower than the modulus of rupture (0.474 ksi) for the concrete topping. Therefore, temperature gradients alone cannot create high enough tensile stresses to cause cracking. However, when the effects of differential shrinkage and temperature gradients are combined then these stresses exceed the rupture stress. Table 1 provides a summary of the stresses created at the top and bottom of the deck respectively. As a result the top of the deck is likely to experience longitudinal cracking above the web of the precast girder due to the combined effects of differential shrinkage and temperature gradient. The bottom of the deck will be subject to transverse and longitudinal cracking. It is important to note that the analysis performed at the cross-sectional level shows stress distributions that apply along the entire bridge superstructure. Therefore, if the tensile stresses in the deck are higher than its modulus of rupture, cracks could potentially develop along the entire bridge length and width.



Fig.10 (a) Stress distribution - negative temperature gradient, (b) Stress distribution - positive temperature gradient

Tensile stresses at the top of the deck						
Section	Differential	Temperature	Total (ksi)	f _r (ksi)	Total/f _r	
	Shrinkage (ksi)	Gradient (ksi)				
1	0.052	0.15	0.202	0.474	0.43	
2	0.352	0.154	0.506	0.474	1.07	
3	-0.264	0.15	-0.114	0.474	N/A	
Tensile stresses at the bottom of the deck						
Section	Differential	Temperature	Total (ksi)	f _r (ksi)	Total/f _r	
	Shrinkage (ksi)	Gradient (ksi)				
1	0.373	0.1	0.473	0.474	1.0	
2	0.496	0.09	0.586	0.474	1.24	
3	0.487	0.02	0.507	0.474	1.07	

Table 1. Tensile stresses at the top and bottom of the deck due to differential shrinkage and temperature gradient

TIME DEPENDENT ANALYSIS AT THE STRUCTURAL LEVEL

The indeterminacy of the superstructure plays an important role when it comes to evaluating the effects of differential shrinkage, creep and temperature at the structure level. For example axial contraction as a result of a uniform decrease in temperature in the longitudinal direction of the two-span continuous bridge can cause significant tensile stresses in the topping and in the precast beam if not accommodated. In addition the curvatures created as a result of differential shrinkage and temperature gradients cause axial contractions and expansions, which need to be allowed to take place to reduce the likelihood of developing additional stresses that might lead to excessive cracking. The following discussion illustrates some of the effects that these phenomena can have if the bearing details at the abutments do not allow axial movements.

Another type of restraint at the structure level in multi-span bridges is the moment restraint at the intermediate supports. The restraint moments develop because the curvatures created by the differential shrinkage and temperature gradients are not allowed to freely take place due to the continuity of the bridge at the interior supports. The assumptions made at the structural level to perform a time dependent analysis are as follows:

- The axial restraint provided by the abutments in the longitudinal direction was assumed to be rigid.
- Plane sections remain plane
- Sections are un-cracked
- Creep and shrinkage properties are assumed to represent the average behavior of the whole cross-sections, or components thereof, in drying conditions.
- Tensile creep is the same as compressive creep

Axial Restraint at the Abutments (Differential Shrinkage, Temperature Gradient, Uniform Temperature)

In a statically determinate structure the bridge superstructure will be free to contract and expand axially and therefore there will be only axial strains and no stresses. However, if this axial movement is restrained, the restraining axial force will create significant stresses in the superstructure. The calculation of these stresses can be performed by imposing the principle of compatibility that requires the total axial deformation to be zero at all bearings where this deformation is restrained. The restraining force can be calculated using the force method of structural analysis in which the axial deformation due to differential shrinkage, temperature gradient and uniform temperature changes must be equal to the axial deformation caused by the restraining force. The uniform temperature change used in this investigation was based on 2010 AASHTO LRFD Bridge Design Specifications¹¹ and was calculated to be 70°F. Figure 11(a) shows the stress distribution in the composite crosssection due to axial restraint at the abutments. The axial stresses due to axial restraint against negative temperature gradient in the topping and the precast beam are small (0.042 ksi and 0.06 ksi, respectively). However, the axial stresses due to axial restraint against differential shrinkage and a uniform decrease in temperature are at least 35% greater than the modulus of rupture (0.641 ksi and 0.906 ksi due to differential shrinkage and 0.965 ksi and 1.501 ksi due to a uniform decrease in temperature). The sensitivity of the stress in deck to the shrinkage and creep properties of the deck is illustrated in Figure 11(b). The horizontal and vertical lines represent the modulus of rupture and ultimate shrinkage strain for the deck, respectively. If the creep coefficient is assumed to be $\varphi = 2.0$ then there is a 136 psi decrease in the tensile stress in the deck for every 100 $\mu\epsilon$ decrease in shrinkage strain.



Fig.11 (a) Stress distribution in Section 1 due to potential axial restraint at the abutments in the longitudinal direction; (b) Sensitivity of stress in deck to shrinkage and creep properties of the deck

Similarly, if the free ultimate shrinkage strain of the deck is assumed to be $\epsilon = -500 \times 10^{-6}$, then there is an 81 psi decrease in the tensile stress in the deck for every increase by 0.5 in the creep coefficient.

As a result, to reduce the likelihood of excessive transverse cracking, axial movement in the longitudinal direction should be accommodated at the abutments. If this movement is restrained, then a topping mix with low shrinkage and high creep will help reduce the tensile stresses. Tensile stresses developed as a result of axial restraints at the abutments in the longitudinal direction due to differential shrinkage, negative temperature gradient and a uniform decrease in temperature apply not only to the entire bridge superstructure but are also constant throughout the depth of the cross-sections. These high tensile stresses have the potential to develop full depth transverse cracks. In addition to the obvious serviceability and durability problems that these high tensile stresses can create, full depth cracks in regions of small moment can cause reductions in shear strength.

Figure 12(a) shows how the stress in the deck due to a uniform decrease in temperature is affected by creep and aging coefficients. The calculation of the restraining axial force at the abutments due to a uniform change in temperature was based on Equation (17). This equation was derived based on the principle of deformation compatibility and the fact that the deck concrete will creep and age whereas the precast girder has already aged and crept when continuity is established. For a fixed aging coefficient of 0.7, the higher the creep coefficient the lower the tensile stress. The tensile stress values in the deck vary from 1.5 ksi when the creep coefficient is zero to 0.97 ksi when the creep coefficient is 2.0. The corresponding values for the precast beam are 2.32 ksi and 1.5 ksi, respectively. Similarly, for a fixed creep coefficient in the deck equal to 2.0, the higher the aging coefficient the lower the tensile stress. This is illustrated in Figure 12(b). The tensile stress values in the deck vary from 0.965 ksi when the aging coefficient is 0.7 to 0.89 ksi when the aging coefficient is 1.0. The corresponding values for the precast beam are 1.5 ksi and 1.384 ksi respectively. This highlights the advantage of a concrete mix that has high creep and does not age significantly. The influence of a higher creep coefficient is more pronounced in reducing the tensile stresses in the deck and the precast beam compared to the aging coefficient. Consequently priority should be given to a mix that has high creep.

$$N_{restraint} = \frac{\alpha \Delta T_{uniform} E_G A_{transformed}}{1 + \frac{A_D \mu_D \varphi_D}{A_D + A_G (1 + \mu_D \varphi_D)}}$$
(17)

where:

 $\begin{aligned} &\alpha = \text{coefficient of thermal expansion} \\ &\Delta T_{\text{uniform}} = \text{uniform change in temperature} \\ &E_G = \text{modulus of elasticity of the precast inverted T} \\ &A_{\text{transformed}} = \text{transformed area of the composite section} \\ &A_D = \text{area of the deck} \\ &\mu_D = \text{aging coefficient for deck concrete} \\ &\varphi_D = \text{creep coefficient for deck concrete} \\ &A_G = \text{area of precast inverted T} \end{aligned}$



Fig.12 (a), (b) Sensitivity of stress in deck and in the precast inverted T-beam due to a uniform decrease in temperature to creep coefficient and aging coefficient, respectively

It should be noted that in the analysis performed in this study the axial restraining stiffness of the abutments was taken equal to infinity. Additionally, the creep coefficient for the precast beams was taken equal to zero because, as stated earlier, it is believed that if the age of continuity is at least 90 days most of the creep in the precast beams has already taken place. In reality the axial restraining stiffness of the abutments will be smaller than infinity and the creep coefficient for the precast beam will be higher than zero. As a result, the stresses created in the deck and the precast beam may be slightly lower than the values presented in this paper.

Moment Restraint at the Intermediate Support

Restraint moments at the intermediate supports are another source for developing tensile stresses in the deck that can lead to excessive transverse cracking. These moments are developed as a result of the restraint to the curvatures induced by creep of concrete under sustained loads and prestressing, differential shrinkage and temperature gradients. The calculation of restraint moment (M_r) due to prestressing, sustained loads and differential shrinkage is based on Equation (18) (Peterman and Ramirez¹⁴):

$$M_{r} = \left(\frac{3}{2} \alpha M_{p} - \alpha M_{dprecast}\right) \left[\Delta (1 - e^{-\varphi_{1}})\right] - \alpha M_{dCIP} \left(1 - e^{-\varphi_{2}}\right) - \frac{3}{2} \alpha M_{s} \left(\frac{1 - e^{-\varphi_{2}}}{\varphi_{2}}\right)$$

$$(1 - e^{-\varphi_{2}}) - \frac{3}{2} \alpha M_{s} \left(\frac{1 - e^{-\varphi_{2}}}{\varphi_{2}}\right)$$

$$(1 - e^{-\varphi_{2}}) - \frac{3}{2} \alpha M_{s} \left(\frac{1 - e^{-\varphi_{2}}}{\varphi_{2}}\right)$$

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$$(1 - e^{-\varphi_{2}}) - \frac{3}{2} \alpha M_{s} \left(\frac{1 - e^{-\varphi_{2}}}{\varphi_{2}}\right)$$

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where:

 M_p = moment caused by prestressing force about centroid of composite member M_s = differential shrinkage moment

 $M_{dnrecast}$ = mid-span moment due to dead load of precast members

 M_{dCIP} = mid-span moment due to dead load of CIP topping

 ϕ_1 = creep coefficient for creep effects initiating when prestress force is transferred to the precast panels

 ϕ_2 = creep coefficient for creep effects initiating when CIP topping is cast

 α = factor that accounts for the relative flexural stiffnesses of the spans and diaphragm

 Δ (1-e^{- ϕ_1}) = change in expression (1-e^{- ϕ_1}) occurring from time CIP topping is cast to time corresponding to restraint moment calculation.

The first term represents the restraint moment due to creep of the precast member due to prestressing force and the weight of the precast member. The second term represents the restraint moment due to creep of the precast member due to the cast-in-place topping weight. The third term represents the restraint moment due to differential shrinkage. Peterman and Ramirez¹⁴ provide additional information for the calculation of some of the terms defined above including an equation for the calculation of differential shrinkage moment. However, this equation does not account for the restraint provided by steel in the precast member and shrinkage induced creep in precast and cast-in-place components. As a result the calculation of differential shrinkage moment was based on Menn's method¹⁰, which considers all the aforementioned effects. Table 2 provides a summary of differential shrinkage moments calculated using various methods. The method proposed by Peterman and Ramirez is simple to use and estimates a differential shrinkage moment which is only 11% different from the one calculated using Menn's Method. PCA method provides a much conservative estimate of the differential shrinkage moment. This is a result of the fact that PCA Method does not correctly account for the restraining effect that the precast girder and reinforcing steel has on the free shrinkage of the deck. In this paper only one time step was used to calculate the differential shrinkage moment using the CTL method. Additional information on the calculation of differential shrinkage moment based on the methods mentioned above is provided in Reference 10 and 14-17. The calculation of restraint moments due temperature gradients was based on Gilbert¹³.

Method	Differential shrinkage	% difference with Menn's		
	moment (ft-kips)	Method		
Menn ¹⁰	646			
PCA ^{15,16}	1393	216%		
CTL ¹⁷ *	724	12%		
Peterman and Ramirez ¹⁴	714	11%		

Table 2. Summary of differential shrinkage moments using various methods

Note: * Using only one time step

Tensile stresses developed as a result of moment restraint due to differential shrinkage and negative/positive temperature gradients are maximum at the intermediate support and reduce linearly towards the abutments (for a two-span continuous bridge). Because a positive temperature gradient causes a positive restraint moment at the intermediate support its effects were not investigated because the focus of this paper was potential cracking on the top surface of the deck. As stated earlier the analysis performed in this study assumes and age of continuity equal to at least 90 days, which represents a best case scenario for reducing positive restraint moments, and a worst case scenario for developing negative restraint moments.

Figure 13(a) shows the stresses in the composite cross-section due the negative restraint moments caused by differential shrinkage and negative temperature gradients. The corresponding maximum tensile stresses in the deck are 1.291 ksi and 0.145 ksi, respectively. The stresses at the top of the precast inverted T-beam due to negative temperature gradient and differential shrinkage are 0.098 ksi and 0.87 ksi, respectively. Table 3 provides a summary of these values as well as the ratio of the total tensile stress due to negative temperature gradient and differential shrinkage to the modulus of rupture. The tensile stresses created as a result of negative temperature gradient are smaller than the modulus of rupture for the deck (0.474 ksi), whereas those created from differential shrinkage are more than 2.7 times. It can be seen that the sum of negative restraint moments creates tensile stresses in the deck and precast inverted T-beam that are well past the modulus of rupture.

	Negative Temperature Gradient	Differential Shrinkage	Total	f_r	Total/f _r
Stress at the top of the deck (ksi)	0.145	1.291	1.436	0.474	3.03
Stress at the top of precast (ksi)	0.098	0.87	0.968	0.671	1.44

 Table 3. Stresses due to negative restraint moments

Figure 13(b) shows the sensitivity of the maximum tensile stress in deck to the shrinkage and creep properties of the deck. The horizontal and vertical lines represent the modulus of rupture and the ultimate shrinkage strain for the deck, respectively. It can be seen that while the maximum tensile stress in the deck is sensitive to the ultimate shrinkage strain, it is not that sensitive to the creep coefficient of the deck. If the creep coefficient is assumed to be 2.0 than there will be a 275 psi decrease in the tensile stress for every 100 $\mu\epsilon$ reduction in the free ultimate shrinkage strain of the deck.



Fig.13. (a) Stress distribution in Section 1 due to differential shrinkage/creep and negative temperature gradient; (b) Sensitivity of tensile stress at the top of the deck to shrinkage and creep properties of the deck.

The negative moments due to superimposed dead and live loads at service for the U.S.360 Bridge are 107 kip-ft. and 219 kip-ft., respectively. The restraint moment due to differential shrinkage and shrinkage induced creep is 909 kip-ft, which is nearly 2.8 times greater than the sum of the negative moments due to dead and live loads. The negative restraint moment due to negative temperature gradient is 102 kip-ft, which is slightly lower than the negative moment due to superimposed dead loads. Table 4 summarizes the magnitudes of negative moments at the interior support. This highlights the significance of negative restraint moments developed as a result of time dependent effects in terms of magnitude. Menn¹⁰ states in his book "Prestressed Concrete Bridges" that: "Theoretically no sectional forces are present at the ultimate limit state due to restrained deformations in ductile systems. In general, restrained deformations are significant only for the behavior of structures under service load conditions with regards to cracking and deflections". This is due to the fact that a ductile system can accommodate imposed curvatures and axial strains by the formation of plastic hinges and yielding of the reinforcing steel. Consequently, while these high restraint moments do not present a safety concern they do need to be controlled to reduce the likelihood of excessive cracking. In this regard, specifying an optimized age of continuity in which the competing effects of negative and positive restraint moments would cancel each other as much as possible is essential. High positive restraint moments negate the effects of negative live load moments and may render a continuous design even more expensive than a design based on simply supported beams. High negative moments may create excessive cracking on the bridge decks and reduce the service life of bridges.

Load Cases	Negative Moments (ft-kips)			
Superimposed Dead Load	107 (M _{negsuperDL})			
Superimposed Live Load	219 (M _{negsuperLL})			
Shrinkage + Creep	909 (M _{negSH+CR})			
Negative Temperature Gradient	102 (M _{negTG})			
Ratios				
$(M_{negSH+CR})/(M_{negsuperDL} + M_{negsuperLL}) = 2.8$				
$(M_{negTG})/(M_{negsuperDL} + M_{negsuperLL}) = 0.3$				

Table 4. Negative moments at interior support

CONCLUSIONS AND RECOMMENDATIONS

This paper has demonstrated that time dependent effects can cause significant stresses in composite concrete bridge superstructures with precast inverted T-beams. To reduce the likelihood of excessive cracking recommendations are made in the following five frameworks. While these recommendations were based on the analysis of a two-span continuous bridge with precast inverted T-beams, most of them apply to most types of composite bridges.

- Mix Design – It is recommended that the concrete mix for the CIP topping possesses low shrinkage and high creep, as these properties help relax tensile stresses built-up due to time dependent effects. While it may be onerous to the supplier to conduct creep tests for various mix designs, it is relatively simple to conduct short term shrinkage tests (at 28 days) on a variety of mixes. This will help create a database of shrinkage values for various mixes that could be used in future projects if the specifications require a mix with certain shrinkage parameters. The engineer of record can use one the shrinkage models available in AASHTO or ACI 209 "Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete" to relate the ultimate shrinkage strain to a strain at 28 days. A mix design for the cast-in-place concrete topping with low shrinkage and high creep is also recommended for composite bridges with steel girders. Because structural steel does not shrink or creep the effects of differential shrinkage and creep would be as pronounced as in the case considered in this paper when the age of continuity for the precast beams is assumed to be 90 days.
- **Cross-Sectional Shape** Inverted T-beam system reduces the tensile stresses in the CIP topping as a result of differential shrinkage compared to voided slabs and adjacent box girders by providing a smaller moment arm between the centroid of cast-in-place topping and that of the precast beam.
- **Mild steel** While mild steel in the cast-in-place topping restrains its free shrinkage, tensile stresses in the CIP topping are not greatly influenced by the amount of mild steel in the topping. Mild steel needs to be provided to control the cracks and help distribute live loads in the transverse direction.
- **Boundary Conditions** In two-span continuous bridges it is essential accommodate axial movement in the longitudinal direction at the abutments to avoid tensile stresses

in the deck due to differential shrinkage, negative temperature gradients and uniform decreases in temperature.

• Age of Continuity –It is recommended that the age of continuity is selected such that the competing effects of positive and negative restraint moment cancel each other as much as possible. High positive restraint moments negate the effects of negative live load moments and may render a continuous design even more expensive than a design based on simply supported beams. High negative moments may create excessive cracking on the bridge decks and reduce the service life of bridges.

COMMENTS

- It is important to note that the analysis performed at the cross-sectional level shows stress distributions that apply along the entire bridge superstructure. Because the precast inverted T-beams serve as stay-in-place forms for the cast-in-place topping, there are no stresses in the cast-in place topping due to its self-weight. Accordingly, any tensile stresses created in the topping due to time dependent effects could not be counter balanced by any compressive stresses due to the self-weight of the topping as those stresses apply only to the precast beams. Therefore, if the tensile stresses in the deck are higher than its rupture stress, cracks could potentially develop in the entire top surface of the bridge in the longitudinal and transverse directions
- Similarly, tensile stresses developed as a result of axial restraints at the abutments in the longitudinal direction due to differential shrinkage, negative temperature gradient and a uniform decrease in temperature apply not only to the entire bridge superstructure but are also constant throughout the depth of the cross-sections. These high tensile stresses have the potential to develop full depth transverse cracks. In addition to the obvious serviceability and durability problems that these high tensile stresses can create, full depth cracks in regions of small moment can cause reductions in shear strength.
- Tensile stresses developed as a result of moment restraint due to differential shrinkage and negative/positive temperature gradients are maximum at the intermediate support and reduce linearly towards the abutments (for a two-span continuous bridge).

In summary, time dependent effects can cause stresses that are higher than the ones created due to mechanical loads and need to be considered in the analysis and design of bridges to prolong their longevity.

NOTATION:

 A_D = area of cast-in-place deck

 A_G = area of precast girder

 A_{ps} = area of prestressing strands

 A_s = area of mild steel

a = distance between the centroid of cast-in-place deck and centroid of precast girder.

 a_D = distance between the centroid of the cast-in-place deck and centroid of composite section

 a_G = distance between the centroid of the girder and centroid of the composite section

c.g. = center of gravity (centroid)

 E_D = modulus of elasticity of the cast-in-place deck

 E_G = modulus of elasticity of the precast girder

 E_s = modulus of elasticity of mild steel

e = eccentricity of the prestressing force with respect to the centroid of the precast girder

 f'_c = specified compressive strength of concrete for use in design (ksi)

 I_D = moment of inertia of the cast-in-place deck

 I_G = moment of inertia of the precast girder

y = distance from centroid

 $y_{Dbottom}$ = distance from the centroid of the deck to the bottom of the composite section $y_{Gbottom}$ = distance from the centroid of the precast girder to the bottom of the composite section

 α = coefficient of thermal expansion

 $\Delta \varepsilon_D$ = change in strain at the centroid of deck due to time dependent effects

 $\Delta \varepsilon_G$ = change in strain at the centroid of girder due to time dependent effects

 $\Delta \varepsilon_s$ = change in strain in mild steel due to time dependent effects

 ΔX = change in curvature due to time dependent effects

 ΔN = change in axial force due to restrained stress as a result of temperature gradient

 ΔM = change in moment due to restrained stress as a result of temperature gradient

 ΔN_D = change in axial force in the deck due to time dependent effects

 ΔN_G = change in axial force in the girder due to time dependent effects

 ΔN_s = change in force in mild steel due to time dependent effects

 ΔM_D = change in moment in the deck due to time dependent effects

 ΔM_G = change in moment in the girder due to time dependent effects

 $\Delta \sigma_D$ = change in stress in deck due to time dependent effects

 $\Delta \sigma_G$ = change in stress in precast girder due to time dependent effects

 $\Delta \sigma_s$ = change in stress in mild steel due to time dependent effects

 ΔT_1 = change in temperature at location 1 due to temperature gradient

 ΔT_2 = change in temperature at location 2 due to temperature gradient

 ε_1 = free strain at location 1 due to temperature gradient

 ε_2 = free strain at location 2 due to temperature gradient

- ε_{SHD} = ultimate shrinkage strain of the deck
- ε_{SHG} = ultimate shrinkage strain of the precast girder
- φ_D = creep coefficient for the deck
- φ_G = creep coefficient for the precast girder
- μ = aging coefficient
- σ_s = free stress/restrained stress in steel due to temperature gradient
- σ_{d1} = free stress/restrained stress in deck at location 1 due to temperature gradient
- σ_{d2} = free stress/restrained stress in deck at location 2 due to temperature gradient
- σ_a = free stress/restrained stress at the top of precast girder due to temperature gradient

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